

Shadow Rates, Forward Guidance, and Unconventional Monetary Policy*

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Abstract

A shadow rate is often used to measure the overall stance of unconventional monetary policy. However, this paper shows that, by extracting the short end only, the shadow rate omits the rest of the information contained in the term structure, which, as a whole, has a profound impact on the economy. Hence the shadow rate is not a sufficiently informative measure of unconventional monetary policy. To better trace out unconventional monetary policy innovations and measure their corresponding macroeconomic impact, I develop a Forward-Guidance-Corrected Factor-Augmented Vector Auto-Regression (FGC-FAVAR) model which employs forward guidance information contained in survey data on expected lift-off dates. I use this forward guidance data as a proxy for omitted term structure information. After controlling for expectations about lift-off, I find that post-crisis expansionary monetary policy is much more aggressive and effective than that estimated by a standard FAVAR model (Wu and Xia (2016)). I estimate that the efforts by the Federal Reserve to stimulate the economy since July 2009 succeeded in making the unemployment rate in December 2013 3.8 percentage points lower, and the real output 15.6% higher, both of which are larger than the estimates reported in the literature.

1 Introduction

The federal funds rate had been the main policy instrument of the Federal Reserve until the recent global financial crisis in 2008. After the crisis, the federal funds rate has reached its zero lower bound (ZLB)¹, and further decreasing the federal funds rate to stimulate the economy has become

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¹The lower bound is not necessarily zero. For example, the lower bound for the federal funds rate is about 25 bps. But for convenience, I still refer to the lower bound as the “ZLB”.

impossible. As a result, the Federal Reserve turned to unconventional monetary policy measures; i.e., large-scale asset purchases (LSAP) — also commonly called quantitative easing (QE). With these new policy tools, two questions arise naturally: First, how can we summarize the overall stance of unconventional monetary policy? Second, how can we measure the impact of unconventional monetary policy on the macroeconomy?

There are several different methods for measuring unconventional monetary policy. For instance, Cúrdia and Woodford (2011), Chen, Cúrdia, and Ferrero (2012), Gertler and Karadi (2013) use dynamic stochastic general equilibrium (DSGE) models to explain the transmission mechanism of LSAP and estimate its macroeconomic effect. This structural-model literature suggests using the central bank's balance sheet as the policy measure. The problem with this method is, for any given balance sheet volume, the macroeconomic impact varies a lot, critically depending on the theoretical assumptions and could be quite far away from the real world (Cúrdia and Woodford (2011)).

A more popular method is to estimate a shadow rate using a term structure model, and the shadow rate is allowed to be negative. See, for example, Krippner (2012, 2013), Bauer and Rudebusch (2013, forthcoming), Lombardi and Zhu (2014), Christensen and Rudebusch (2015, 2016), and Wu and Xia (2016).

In this paper, I examine how the U.S. macroeconomy has reacted to unconventional monetary policy in the post-crisis period (July 2009² - December 2013), using not only the shadow rate, but also forward guidance information contained in survey data on expected lift-off dates.

To do so, I first show that unconventional monetary policy manipulates the entire yield curve, rather than merely the short end. The shadow rate, however, by extracting the short end only, omits the rest of the information contained in the term structure, which, as a whole, has a profound impact on the economy. Hence the shadow rate is not a sufficiently informative measure of unconventional monetary policy. I provide two pieces of evidence: (1) While the term structure model from which we extract the shadow rate can forecast an expected lift-off date series quite similar to the survey data; a standard FAVAR model that uses the very same shadow rate as the only policy measure fails to forecast similar expected lift-off dates. (2) A standard FAVAR model that uses the shadow rate as the only policy measure delivers a counterintuitive macroeconomic effect estimation.

To better trace out unconventional monetary policy innovations and measure their corresponding macroeconomic impact, in addition to the shadow rate, I introduce forward guidance information contained in survey data on expected lift-off dates, which is used as a proxy for omitted term structure information. I also develop a forward-guidance-corrected factor-augmented vector autoregression (FGC-FAVAR) model to incorporate this forward guidance information.

After controlling for expectations about lift-off, I am able to decompose the (biased) FAVAR model monetary policy innovation $\hat{\epsilon}_{t+1}^{MP}$ into two components: (i) a forward guidance term v_t^{MP} that measures the information given out by period- t forward guidance, known at period t , and (ii) the model innovation ϵ_{t+1}^{MP} that is estimated by the FGC-FAVAR model.

I compare the FGC-FAVAR policy shocks $\{\epsilon_{t+1}^{MP}\}_t$ with the FAVAR policy shocks $\{\hat{\epsilon}_{t+1}^{MP}\}_t$, and show that the standard FAVAR model confounds the period- t forward guidance information with the policy shock, and hence severely underestimates the aggressiveness of unconventional monetary

²Following NBER Business Cycles <http://www.nber.org/cycles.html>, the most recent global financial crisis started at December 2007, and ended at June 2009.

policy in the post-crisis period. My analysis also indicates a status quo of U.S. unconventional monetary policy: On one hand, for a long time, the market always expects that lift-off is just around the corner (within a year or so). On the other hand, the Federal Reserve tries to convince the market that the Fed has shifted to a more accommodative policy approach (a much later lift-off date) by surprising the market with contemporaneous monetary easing.

I then conduct a counterfactual analysis to measure the effect of unconventional monetary policy by simulating the paths of major economic variables if unconventional monetary policy innovations were shut off. I find that the counterintuitive macroeconomic effect estimation that was found in a standard FAVAR model disappears in my FGC-FAVAR model. The efforts by the Federal Reserve to stimulate the economy since July 2009 succeeded in making the unemployment rate in December 2013 3.8 percentage points lower (otherwise the unemployment rate would be as high as 10.5%), and the real output 15.6% higher.

My paper adds to a growing and active literature on the macroeconomic effects of unconventional monetary policy. While the literature generally agrees that unconventional monetary policy is effective, there is no consensus as to how great this effect should be. Chen, Cúrdia, and Ferrero (2012) find QE2 has increased output by 0.1% after 6 quarters. Chung et al. (2012) estimate that QE1 boosted the level of real GDP by almost 3% by late 2012, rendered the unemployment rate 0.75 percentage points lower than it would otherwise have been. Without QE1, Baumeister and Benati (2013) suggest real GDP would have been 0.9% lower, and unemployment would have been 0.75 percentage points higher, reaching 10.6% by the end of 2009. Gertler and Karadi (2013) find the output would have been 3.5% lower relative to the case without QE1. Engen, Laubach, and Reifschneider (2015) show that the LSAP successfully lowered the U.S. unemployment rate by 1.25 percentage points by early 2015. According to the estimation of Weale and Wieladek (2016), the LSAP up to late 2013 leads to a statistically significant rise of 11.7% (confidence interval [5.6%,16.5%]) in real GDP for the U.S.³.

When compared in the same time frame, my results are comparable to Baumeister and Benati (2013)⁴, Gertler and Karadi (2013)⁵, and Weale and Wieladek (2016)⁶, but are significantly larger than the estimates reported in the rest of the literature. One possible reason is that most existing DSGE models assume market participants perfectly anticipates future monetary policy. According to Gertler and Karadi (2013) and Engen, Laubach, and Reifschneider (2015), the effects of the LSAPs depend heavily on the changes in policy expectations. In the real world, the Fed has implemented unconventional monetary policy with great caution. One typical example is that the Fed carefully revised the expected lift-off date from within a year or so to late 2013, and eventually guided the market expectation to late 2015. In early 2009, it is impossible even for the Fed itself to imagine a ZLB period for as long as 6 years. In an exercise that examines the effect of market expectation, I assume a perfect anticipation of lift-off date (December 2015) throughout, and find that the macro

³Weale and Wieladek (2016) finds that an asset purchase of 1% of GDP leads to a statistically significant rise of 0.58% rise in the U.S. real GDP. Using different identification schemes for the Bayesian VAR, this number varies from 0.28% to 0.82%, indicating an overall rise in the real output between 5.6% and 16.5%, and the mean is 11.7%.

⁴Without LASP, by the end of 2009, Baumeister and Benati (2013) estimate that the output would be 0.9% lower, and unemployment rate would be 10.6%; while my estimates are 1% and 10.5%, respectively.

⁵Gertler and Karadi (2013) find the output would have been 3.5% lower relative to the case without QE1; while my estimate is 3.3%.

⁶According the estimation of Weale and Wieladek (2016), the LSAP up to December 2013 leads to a statistically significant rise of 11.7% (confidence interval [5.6%,16.5%]) in real GDP for the U.S.; while my estimate is 15.6%.

effect of LSAP estimated by the FGC-FAVAR model would be much smaller.

The rest of the paper is organized as follows. Section 2 lays out the background. Section 3 describes the data. Section 4 presents the traditional FAVAR model and shows how the shadow rate fails to capture forward guidance information. Section 5 proposes a FGC-FAVAR model that incorporates forward guidance information into the FAVAR framework, and analyzes the macroeconomic impact of unconventional monetary policy. Section 6 concludes.

2 Shadow Rate and Traditional FAVAR

Monetary policy of central banks is traditionally conducted through open market operations to guide the short-term interest rates. For example, the Federal Open Market Committee (FOMC) determines the target federal funds rate, which is an overnight rate at which depository institutions (banks and credit unions) lend reserve balances to each other. By raising and lowering the federal funds rate (see Figure 1), the Federal Reserve can influence a broader range of interest rates, which in turn affects the aggregate demand, including consumer spending, purchases of homes, capital investments by firms, etc, and can help cool down or stimulate the economy.

After the crisis in 2008, because of the ZLB, the Federal Reserve turned to LSAP (see Table 1). The economic logic is intuitive: with the Fed purchasing large amounts of Treasury and government-sponsored enterprise (GSE) mortgage-related securities, long-term rates decrease significantly, which helps stimulate the economy, just as they do under conventional monetary policies. Reduced availability of Treasury and GSE securities also leads investors to purchase other assets, such as corporate bonds, lowering the yields on those assets as well, and further promoting economic recovery.

Per our first question: how can we summarize the overall stance of unconventional monetary policy? As I argued in Section 1, the shadow rate is not only more robust than the central bank's balance sheet, but more importantly, it makes unconventional monetary policy comparable to the conventional measure (federal funds rate). For example, Figure 2 displays the effective federal funds rate and the shadow rate estimated by Wu and Xia (2016). It is useful to compare unconventional monetary policy with conventional monetary policy. The Federal Reserve has been using conventional monetary policy (federal funds rate) for decades, and generally followed the Taylor rule during the terms of Paul Volcker (1979-1987) and Alan Greenspan (1987-2006) (see Clarida, Galí and Gertler (2000)). The Fed has plenty of experience under various circumstances that helps it discern when to change the federal funds rate, and determine how much the adjustment would be desirable. On the contrary, the usage of LSAP is unprecedented, which brings a lot of uncertainty, and makes the Fed extremely cautious in this process⁷. If the shadow rate is indeed a sufficient metric to summarize unconventional monetary policy, then it would be easier than using the central bank's balance sheet to evaluate whether an ongoing QE program has been too hawkish or dovish.

Once we show the shadow rate is a preferred metric (compared to central bank's balance sheet) to (at least partly) model unconventional monetary policy at ZLB, it is then straightforward to use the FAVAR model proposed by Bernanke, Boivin, and Elias (2005) to study the effects of monetary policy.

⁷From Table 1, we can see the Fed has revised the QE programs a couple of times. In addition, the Fed revised (postponed) lift-off date several times (see Table 2).

3 Data

This section describes the data used to estimate the traditional FAVAR model and FGC-FAVAR model.

For ease of comparison, I use the same macroeconomic dataset as Wu and Xia (2016), which includes 97 monthly data series⁸ from January 1960 to December 2013. These macroeconomic data are from the IHS Global Insight Database. Detailed data description can be found here: Wu and Xia (2016) Online Appendix.

I also use Wu and Xia (2016)'s shadow rate from January 2009 to December 2013, which was estimated from a nonlinear shadow rate term structure model using one-month forward rates with 3-month, 6-month, 1-year, 2-year, 5-year, 7-year, and 10-year maturities.

I set a lower bound of the federal funds rate $\underline{r} = 0.25\%$, following Wu and Xia (2016).

3.1 ZLB Duration and Lift-off Date

The ZLB duration $N(t)$ measures the market's perception of when the economy will finally escape from the ZLB. This is a random variable. Given the information at time t , it is defined as:

$$N(t) \equiv \inf\{N(t) \geq 0 | s_{t+N(t)} \geq \underline{r}\} \quad (1)$$

where $s_{t+N(t)}$ is the shadow rate at time $t + N(t)$. Thus $N(t)$ represents how much (expected) time passes before the shadow rate first crosses the lower bound from below, and $t + N(t)$ is the (expected) lift-off date. At time t , $s_{t+N(t)}$ is unknown. I will provide and compare three sets of lift-off dates, denoted as:

- $\{t + N(t)\}_t$: The market participants' expected lift-off date time series (survey data) is from Blue Chip Financial Forecast and New York Fed Primary Dealer Survey, following Williams (2014)⁹. Figure 3 gives a sample survey question and the answer for the expected lift-off date $t + N(t)$ (3rd quarter 2014¹⁰) at period t (March 2012).
- $\{t + \hat{N}(t)\}_t$: The expected lift-off date time series estimated by a standard FAVAR model using the constructed policy rate s_t^o , which is constructed by splicing the federal funds rate r_t before ZLB (before January 2009) and the shadow rate s_t since the ZLB (since January 2009).
- $\{t + \tilde{N}(t)\}_t$: The expected lift-off date time series estimated by Wu and Xia (2016)'s term structure model.

⁸The effective federal funds rate is included in the dataset, but it is replaced by the constructed policy rate right away; i.e., replaced by the rate constructed by splicing the effective federal funds rate before 2009, and the shadow rate since 2009. Yield spreads of 3-month, 6-month, 1-year, 5-year, 10-year maturity rates against the federal funds rate are also included in the dataset, but are replaced right away by the yield spreads of different maturity rates against the constructed policy rate.

⁹A detailed description can be found in Appendix A.1.

¹⁰Following Wu and Xia (2016), I interpret this date into the first month of this quarter. That is, July 2014. In a separate unreported exercise, I tried other translation methods, including the middle month of this quarter (in this example, August 2014), and the latest month of this quarter (in this example, September 2014). The robustness check shows that, my estimation of the macro impact does not change much when the expected lift-off date changes for a month or two.

4 Traditional FAVAR

This section is organized as follows: Section 4.1 presents a standard FAVAR model using the policy rate s_t^o . Section 4.2 uses the FAVAR model to estimate the macroeconomic effect of unconventional monetary policy, and show that it delivers counterintuitive estimation. Section 4.3 estimates the expected lift-off time series $\{t + \hat{N}(t)\}_t$ from the FAVAR model, and compares it with the expected lift-off time series $\{t + \tilde{N}(t)\}_t$ estimated from Wu and Xia (2016)'s term structure model and the survey data on expected lift-off time series $\{t + N(t)\}_t$, showing that the shadow rate omits critical information in the term structure and hence is not a sufficiently informative measure of unconventional monetary policy.

4.1 Model

The basic idea of FAVAR framework is to summarize compactly the rich information contained in a large set of economic variables Y_t using a low-dimensional vector of factors z_t . This model allows us to study monetary policy's impact on any macroeconomic variable of interest. The factor structure also ensures that the number of parameters does not explode. The model is defined as follows¹¹:

$$z_{t+1} = \mu + \rho z_t + \Sigma \epsilon_{t+1} \quad (2)$$

where $z_t = [x_t^1, x_t^2, x_t^3, s_t^o]'$, x_t^1 , x_t^2 and x_t^3 are the 3 factors extracted from the 97 macroeconomic data series Y_t , s_t^o the policy rate constructed by splicing the effective federal funds rate r_t before 2009 and the shadow rate s_t since January 2009; $\mu = [\mu_x^1, \mu_x^2, \mu_x^3, \mu_s]'$ are the constants; ρ is the autoregressive matrix; and Σ is the cholesky decomposition of the covariance matrix. Structural shocks $\epsilon_{t+1} = [\epsilon_{t+1}^1, \epsilon_{t+1}^2, \epsilon_{t+1}^3, \epsilon_{t+1}^{MP}]'$ are assumed to follow standard normal distribution $N(0, I_4)$. Specifically, the 4th dimension ϵ_{t+1}^{MP} is the monetary policy shock.

Observed macroeconomic variables load on the macroeconomic factors and the shadow rate as follows:

$$Y_t = a + b_x^1 x_t^1 + b_x^2 x_t^2 + b_x^3 x_t^3 + b_s s_t^o + \eta_t \quad (3)$$

where Y_t is the 97 by 1 macroeconomic variable vector, a is the 97 by 1 intercept; b_x^1 , b_x^2 , b_x^3 and b_s are all 97 by 1 factor loadings; $\eta_t \sim (0, \Omega)$.

Based on the autoregressive equation (2), we can estimate the expected lift-off date $t + \hat{N}(t)$ given information available at period t . Using equation (3), we can trace out the monetary policy effect on the macroeconomic variables.

4.2 Macroeconomic Effects of Unconventional Monetary Policy

To measure the effects of unconventional monetary policy, an intervention analysis is conducted — i.e., a counterfactual analysis that simulates the paths of major economic variables if unconventional monetary policy innovations were shut off. This approach is in line with Box and Tiao (1975)'s approach with multiple interventions. Since we are using Wu and Xia (2016)'s data and the standard

¹¹For ease of exposition, the FAVAR model is expressed with 1-lag, but it is easy to generalize the model to any p -lag case. In the empirical analysis in this paper, I use 13-lags unless otherwise specified.

FAVAR model, the results in this section are merely a replication of Wu and Xia (2016). Figure 4 cites the comparison of the constructed/realized time series for 6 major macroeconomic variables (in blue), and counterfactual paths (in red) for an alternative world where all the (unconventional) monetary policy shocks $\{\epsilon_t^{MP}\}_t$ at the post-crisis period (from July 2009 to December 2013) were zero.

Figure 4 highlights two counterintuitive observations:

Observation I: The constructed policy rate (blue solid line) is higher than the counterfactual policy rate (red dashed line) until early 2011, indicating that the Fed had adopted a contractionary policy until early 2011. This result contradicts our knowledge that the Fed was attempting to conduct expansionary monetary policy within this period. Indeed, the first round of quantitative easing (QE1) started from November 2008 to March 2010, and QE2 started from November 2010 to June 2011. By early 2011, QE2 is still under way.

Observation II: In the absence of unconventional monetary policy, by December 2013, the unemployment rate would be only 0.13 percentage points higher at the 6.83% level rather than 6.7% in the observed data. The industrial production index would have been 101.0, slightly lower than the observed 101.8, and capacity utilization would be only 0.3% lower than what we observe. Housing starts would be only 11,000 lower (988,000 vs. 999,000). CPI would be slightly higher (235.45 vs. 234.58).

In sum, the counterfactual analysis predicts that unconventional monetary policy has a negligible effect on the macroeconomy, and even in the wrong direction! This is contradictory to the literature summarized in Section 1, which investigates the macroeconomic effect of the LSAP and has found noticeable effects.

What is missing? What could go wrong? One possibility is that the shadow rate is not a sufficiently informative measure of unconventional monetary policy. Note that during the ZLB period, besides the LSAP programs described in Table 1, another important component of unconventional monetary policy conducted by the Fed is constant communication with the public about future monetary policy, the so-called “forward guidance” (see Table 2). The former part — “*what the Fed does*”: QE1, QE2, operation twist, QE3, etc — is (partly) captured by the shadow rate; while the latter part — “*what the Fed says*”: forward guidance — is completely omitted. According to Engen, Laubach, and Reifschneider (2015), both LSAP and forward guidance are parts of unconventional monetary policy, and are highly interdependent, so their macroeconomic effects should be evaluated jointly. I will show how the shadow rate and the traditional FAVAR model omit forward guidance information in Section 4.3.

4.3 Expected Lift-off Date

In Section 4.2, I have shown in detail how the macroeconomic effects of unconventional monetary policy estimated by a standard FAVAR model with a shadow rate as the only policy measure contradict much evidence in the literature as well as our knowledge.

I argue that such counterintuitive results can be attributed to the fact that the shadow rate is not a sufficiently informative measure of unconventional monetary policy. As I explained in Section 1, unconventional monetary policy flattens the entire yield curve and changes the whole term structure,

rather than only manipulating the short end. By extracting the short end only, the shadow rate omits the rest of the information contained in the term structure, which, as a whole, has a profound impact on the economy. The most representative term structure information the shadow rate fails to capture is the likely future course of monetary policy, or to be more specific, the expected lift-off date, which has been the main focus of the Federal Reserve’s forward guidance statements and been acknowledged in the literature (see Chen, Cúrdia, and Ferrero (2012), Engen, Laubach, and Reifschneider (2015), Bauer and Rudebusch (forthcoming)). For example, the post-meeting statement issued in September 2012 noted that the Fed anticipated that “exceptionally low levels for the federal funds rate are likely to be warranted at least through mid-2015” (see Table 2).

To show this, I compare the expected lift-off time series $\{t + \hat{N}(t)\}_t$ from the FAVAR model, with the expected lift-off time series $\{t + \tilde{N}(t)\}_t$ estimated from Wu and Xia (2016)’s term structure model and the survey data on expected lift-off time series $\{t + N(t)\}_t$. If forward guidance information is negligible, or is already captured properly by the shadow rate, then we should expect the FAVAR-predicted lift-off date series $\{t + \hat{N}(t)\}_t$ to be close/similar to the survey data on lift-off date series $\{t + N(t)\}_t$. This comparison is summarized in Figure 5. Figure 5 plots the corresponding expected lift-off date series $\{t + \hat{N}(t)\}_t$ (in green) predicted by the traditional FAVAR, $\{t + \tilde{N}(t)\}_t$ (in blue) predicted by the shadow rate term structure model, and the survey series $\{t + N(t)\}_t$ (in red), respectively. The vertical purple lines are the forward guidance dates, when the Fed unambiguously gave out the dates (in black dot) on which the Fed expected the ZLB to end.

We can draw two conclusions from the comparison of the expected lift-off time series.

- The survey data on market expectation $\{t + N(t)\}_t$ indicate the average expected ZLB duration $\{N(t)\}_t$ is 16 months, and that the market expectation is consistent with the Fed’s forward guidance. This result is generally consistent with findings in Femia, Friedman, and Sack (2013). On the contrary, a traditional FAVAR model predicts an average expected ZLB duration $\{\hat{N}(t)\}_t$ of 67 months, and that the market expectation deviates radically from the Fed’s forward guidance. The FAVAR estimates are wildly implausible. For example, it is hard to believe that in 2009 any market participant ever anticipated that the ZLB period would last until late 2017, or that in early 2011 anyone ever anticipated the ZLB period would last until 2019. Such a dramatic contrast of the survey lift-off series compared with that predicted by the FAVAR model shows that forward guidance contains large amounts of information that the traditional FAVAR model (and the shadow rate) failed to capture. Put another way, it seems possible that the news shocks $\{\epsilon_t\}_t$ in the traditional FAVAR model (2), which is assumed to be purely unanticipated, also impounds forward guidance information. This model mis-specification could lead to severe bias in estimating the model innovations $\{\epsilon_t\}_t$, and the effect of the monetary policy.
- The expected lift-off date $\{t + \tilde{N}(t)\}_t$ predicted by Wu and Xia (2016)’s term structure model is pretty close to the survey data on market expectation and the Fed’s forward guidance, indicating that the term structural model, as a whole, successfully captures forward guidance information. Note that this is not an isolated finding. As a matter of fact, there is plenty of similar evidence in the literature. For example, Bauer and Rudebusch (2013, forthcoming) also find that term-structure-model-implied expected lift-off dates are very closely matched by survey forecasts. The interesting thing is that the shadow rate used in the FAVAR model to estimate the misleading $\{t + \hat{N}(t)\}_t$ is derived from the very same term structure model.

While the term structure model gives out an expected lift-off date that is similar to survey forecasts and the Fed’s forward guidance, the FAVAR model fails to provide similar forecasts, confirming that the shadow rate omits critical information in the term structure and hence is not a sufficiently informative measure of unconventional monetary policy.

5 Forward-Guidance-Corrected FAVAR (FGC-FAVAR)

I have shown in the previous section that the shadow rate and the traditional FAVAR model fail to capture forward guidance information. In this section, I develop what I call a Forward-Guidance-Corrected FAVAR, or FGC-FAVAR for short. This framework exploits the information contained in the survey data on expected lift-off dates $\{t + N(t)\}_t$, and imposes this information as a forward guided path restriction to estimate unconventional monetary policy innovations and effects.

5.1 Forward Guidance: Economic Intuition

In principle, the entire yield curve (i.e., the whole term structure of interest rates) affects the economy. Before the crisis, the Federal Reserve only adjusted the federal funds rate (i.e., the intercept of the yield curve) to influence the macroeconomy. Conditional on the state of the economy, there was a stable relation between short and long term rates. Hence, the federal funds rate was an appropriate metric that summarized the overall stance of conventional monetary policy. However, after the crisis, the Federal Reserve employs LSAP that aims at lowering the long term rates, which effectively flattens the yield curve and changes the whole term structure. Hence, an estimated shadow rate which only contains the short end information of the term structure cannot fully capture unconventional monetary policy. Take the expected lift-off date, for instance. I showed in Section 4.3 that a term structure model (which is used to estimate a shadow rate) can give out an expected lift-off date pretty much in line with the Fed’s forward guidance statements, while a standard FAVAR model that uses the very same shadow rate as the only policy measure predicts wildly implausible lift-off dates.

So far I have made the point that the shadow rate is not a sufficiently informative measure to summarize the overall stance of unconventional monetary policy, and we need to incorporate other term structure information into the analysis. Of course, I have not discussed what other term structure information should be included. There are plenty of parameters of the yield curve: long-term rates of different maturities, slope, curvature, etc. Obviously we cannot include all of them, and we have to make our choices. In this paper, I choose to incorporate the expected lift-off date — the date that the Fed is expected to raise the federal funds rate for the first time — into the analysis. According to Bauer and Rudebusch (forthcoming), the expected lift-off date can provide a useful measure of the stance of monetary policy and the tightness of the ZLB. Chen, Cúrdia, and Ferrero (2012) find that the commitment to keep the short-term interest rate at the ZLB for an extended period increases the response of economic activities. Furthermore, the boost from the commitment to the ZLB is increasingly larger with the length of such a commitment. Also, as we can discern ourselves, the expected lift-off date is the most important (if not the only) information the Fed talks about in a forward guidance statement, and has a considerable effect on the term structure¹².

¹²Of course, the expected lift-off date does not contain certain information of the term structure, for example, the

Forward guidance aims to guide the market’s expectation regarding the future short rate. The Federal Reserve tries to convince market participants that they will keep policy on an exceptionally easy course even beyond the point at which they would normally start to tighten for fear of incipient inflation. The Federal Reserve has made considerable use of forward guidance since the federal funds rate first hit the ZLB. According to Glick and Leduc (2013), there were 46 announcements released by the Federal Reserve between November 2008 and December 2013, almost one announcement per month.

Usually, the Fed does not explicitly give out an expected lift-off date in the announcement¹³, but the wordings of forward guidance generally focus either on (i) the length of the ZLB (vaguely), or (ii) the target unemployment rate and inflation. Femia, Friedman, and Sack (2013) find that the market participants have interpreted the Federal Reserve’s policy guidance as conveying important information about the Fed’s policy reaction function. For example, the market participants had the perception that there was a range of economic outcomes that could be consistent with the indicated lift-off date. Specifically, market participants expected the Fed to wait for lower levels of unemployment for a given level of inflation before beginning to raise the target federal funds rate, thereby shifting to a more accommodative policy approach aimed at supporting the economic recovery.

According to Femia, Friedman, and Sack (2013), the responses to the New York Fed’s Primary Dealer Survey indicate that the market expectation is generally consistent with the Fed’s forward guidance. Hence, I use the market expected lift-off date time series (survey data) as a proxy to summarize the overall stance of the Federal Reserve’s forward guidance¹⁴, and propose a FGC-FAVAR model to incorporate forward guidance information. The above logic is well summarized by Figure 6. The economic intuition is straightforward: With forward guidance about future monetary policy more explicit than ever before, the market participants are supposed to have a better anticipation of future monetary policy as well as the macroeconomy. Using the market expected lift-off date as an additional information source and as a restriction to the econometric model, we intend to isolate and estimate the effect of forward guidance from the news shocks.

$$\hat{\epsilon}_{t+1} = v_t + \epsilon_{t+1} \tag{4}$$

$\hat{\epsilon}_{t+1}$ is the model innovations estimated by the traditional FAVAR method. By using the FGC-FAVAR model, I decompose $\hat{\epsilon}_{t+1}$ into two components¹⁵: (i) v_t measures the information given out by period- t forward guidance, hence it is assumed to be known at period t , and it is estimated by imposing the forward guidance restriction $E_t s_{t+N(t)}^o = \underline{r}$. That is, at period t , the market participants anticipate the Federal Reserve would raise the federal funds rate from the ZLB for the first time in exactly $N(t)$ periods. (ii) ϵ_{t+1} is supposed to be the actual model innovation which describes unconventional monetary policy.

term premium. I argue that the term premium information has already been included in the factors in the FAVAR model, see detailed data description here Wu and Xia (2016) Online Appendix.

¹³There were only 4 times when the Fed explicitly gave out an expected lift-off date, which are listed in Table 2.

¹⁴Bauer and Rudebusch (forthcoming) argue that the pace of expected tightening also matters. However, for the sample period (up until December 2013) considered in this paper, the expected lift-off date from ZLB is an enough good forward guidance summary. The pace of tightening has not become much of an issue for consideration until late 2015.

¹⁵As a matter of fact, Equation (4) is an approximate equation. It does not hold exactly because the traditional FAVAR model and the FGC-FAVAR model have (slightly) different parameters.

Note that traditional unconstrained FAVAR completely omits the information contained in forward guidance, so it confounds the effect of the forward guidance term v_t with the news shocks ϵ_{t+1} . Specifically, when the forward guidance term v_t and the news shock ϵ_{t+1} are in opposite signs, they would offset each other, leading to an underestimation of unconventional monetary policy innovations and their corresponding policy effect, as we observed in Figure 4. I will discuss this in greater detail in Section 5.5 and 5.6.

As a brief review of the whole idea, I find that the traditional FAVAR model predicts that unconventional monetary policy has a negligible effect on the macroeconomy, and even in the wrong direction. Then I suggest there is a bias in the estimated model innovation $\hat{\epsilon}_{t+1}$. The shadow rate and the FAVAR model omit certain information v_t known at period t and confound it with the model innovation ϵ_{t+1} . With forward guidance information contained in survey data on market expected lift-off dates, I succeeded in uncovering a more precise model innovation ϵ_{t+1} by decomposing the biased $\hat{\epsilon}_{t+1}$. Then I am able to use the newly-estimated model innovation ϵ_{t+1} and show that unconventional monetary policy is much more aggressive and effective than that estimated by a standard FAVAR model.

Such a methodology is quite similar to Kuttner (2001), where he looks into the impact of target federal funds rate announcement on government bond rates. Conventional wisdom predicts that an increase in the target federal funds rate leads to an immediate increase in market interest rates, but the literature before him finds only elusive evidence. Kuttner (2001) claims that the failure to distinguish between anticipated and unanticipated actions accounts for the apparent lack of a close link. Using the federal funds futures rates to disentangle expected from unexpected policy actions, he finds a strong relationship between surprise policy actions and market interest rates; the response to anticipated actions is small.

5.2 Model

The autoregressive equation is modified as:

$$z_{t+1} = \mu + \rho z_t + \Sigma v_t + \Sigma \epsilon_{t+1} \quad (5)$$

The FGC-FAVAR model is similar to the traditional FAVAR model to a large extent. Unless otherwise stated, all variables and parameters are similarly defined as in (2). The only difference is the added Σv_t term¹⁶, where $v_t = [0, 0, 0, v_t^{MP}]'$ is the forward guidance adjustment term that measures the additional information given out by the Fed through forward guidance at period t . Note that only the money (4th) dimension of v_t is non-zero, indicating the fact that forward guidance (as well as other monetary policies) only affects the monetary policy dimension. The estimation algorithm of v_t will be described in detail in Section 5.4. In short, I translate the market expected lift-off date series $\{t + N(t)\}_t$ into the forward guidance term $\{v_t\}_t$. I do so by imposing a forward guided path restriction of future monetary policy that the forward guidance term v_t pushes up the expected shadow rate toward \underline{r} in $N(t)$ periods, i.e., the v_t term guarantees $E_t s_{t+N(t)}^o = \underline{r}$ for any post-crisis period t .

¹⁶I could estimate Σv_t as a whole, but I did not because I want to make v_t a comparable term to ϵ_{t+1} , and in this way it would be easier to decompose the standard-FAVAR-estimated $\hat{\epsilon}_{t+1}$ into the forward guidance term v_t and the actual model innovation ϵ_{t+1} . It would also help explain the economic intuition and facilitate my further analysis.

For ease of exposition, we define operator $\hat{E}_t(\cdot)$, which is the (biased) expectation operator based on the standard FAVAR model that confounds the forward guidance v_t term with the model innovation ϵ_{t+1} . If we mistakenly use the mis-specified FAVAR model in Section 4.1, then we have

$$\hat{E}_t z_{t+1} = \mu + \rho z_t \quad (6)$$

$$z_{t+1} - \hat{E}_t z_{t+1} = \Sigma v_t + \Sigma \epsilon_{t+1} \quad (7)$$

We also define operator $E_t(\cdot)$, which is the expectation operator based on the FGC-FAVAR model in the current section. Following (5), we have

$$E_t z_{t+1} = \mu + \rho z_t + \Sigma v_t \quad (8)$$

$$z_{t+1} - E_t z_{t+1} = \Sigma \epsilon_{t+1} \quad (9)$$

As explained in the previous sections, with forward guidance about future monetary policy more explicit than ever before, the market participants are supposed to have a better anticipation of future monetary policy. This newly added Σv_t in the FGC-FAVAR model is the term that captures the additional information given out by the Fed through forward guidance. Notice the timing of v_t in (5): the Fed gives out information about future monetary policy, so this information is known at period t , but this monetary policy does not come into effect until period $t + 1$. This is the nature of forward guidance — guide the market participant’s expectations. Comparing (6) and (7) with (8) and (9), we can see that the traditional FAVAR model completely omits forward guidance information, and confounds the v_t term with the news shocks ϵ_{t+1} . This inevitably leads to an inaccurate estimation of model innovations ϵ_{t+1} and the corresponding macroeconomic effect — especially when the forward guidance term v_t and the news shock ϵ_{t+1} are in opposite signs and offset each other.

To complete the FGC-FAVAR model, observed macroeconomic variables still load on the macroeconomic factors and the shadow rate as follows:

$$Y_t = a + b_x^1 x_t^1 + b_x^2 x_t^2 + b_x^3 x_t^3 + b_s s_t^o + \eta_t \quad (10)$$

All variables and parameters are defined similarly as in (3).

5.3 Econometric Interpretation

This section defines the forward guidance term v_t rigorously, and answers the question — how do we interpret v_t ?

During the ZLB period, the monetary transmission mechanism is recovering from the crisis over time, and the Fed is giving out forward guidance information every period. Thus, the most intuitive method to reflect this reality is to allow for time-varying parameters or time-varying function forms in the FAVAR model¹⁷:

- **Pre-Crisis:** Assume there is no structural break before the financial crisis, and the before-crisis $\{z_t\}_t$ evolves as follows:

$$z_{t+1} = \mu + \rho z_t + \Sigma \epsilon_{t+1} \quad (11)$$

¹⁷For ease of exposition, I only list the autoregressive process $\{z_t\}_t$. The loadings of macroeconomic variables Y_t on z_t is trivial.

- **Post-Crisis:** After the financial crisis, suppose there is a structural break in the parameters, or even worse, the evolution of $\{z_t\}_t$ now follows a time-varying function form $f_t(z_t)$ ¹⁸. That is,

$$z_{t+1} = f_t(z_t) + \Sigma\epsilon_{t+1} \quad (12)$$

and this time-varying function form $f_t(\cdot)$ guarantees the expected lift-off date $t + N(t)$; i.e., $E_t s_{t+N(t)}^o = \underline{r}$.

- **Model Mis-specification:** Suppose we mistakenly use the pre-crisis model (11) to fit the post-crisis data, while the actual data generating process is (12). Then we would end up with a biased estimated model innovation, as summarized in Table 3.

$$\hat{\epsilon}_{t+1} = \Sigma^{-1}[f_t(z_t) - (\mu + \rho z_t)] + \epsilon_{t+1} \quad (13)$$

where $\hat{\epsilon}_{t+1}$ is the biased model innovation estimated from the mis-specified FAVAR model, and ϵ_{t+1} is the actual model innovation.

The biased model innovation $\hat{\epsilon}_{t+1}$ consists of two components:

- (i) A term $\Sigma^{-1}[f_t(z_t) - (\mu + \rho z_t)]$ that is actually known at period t .
- (ii) The actual model innovation ϵ_{t+1} .

v_t is defined to be the bias term in $\hat{\epsilon}_{t+1}$ that is known at period t .

Definition 1 $v_t \equiv \Sigma^{-1}[f_t(z_t) - (\mu + \rho z_t)]$

v_t measures the bias generated by using the old dynamic relationship to fit the new data. That is, v_t measures the bias generated by model mis-specification.

It is straightforward to see that the model mis-specification leads to biased estimation of unconventional monetary policy and its corresponding macroeconomic effect. Moreover, the mis-specified FAVAR model cannot guarantee the expected lift-off date $t + N(t)$; i.e., $\hat{E}_t s_{t+N(t)}^o \neq \underline{r}$.

- **Remedy (FGC-FAVAR):** The bias term v_t , by definition, can be estimated by using more information available at period t . The idea of the FGC-FAVAR model is to use the expected lift-off date $t + N(t)$ as a restriction on the path of future monetary policy to identify the v_t term that is known at period t . The detailed estimation method will be described in Section 5.4. Once we have successfully estimated the forward guidance term v_t using the restriction $E_t s_{t+N(t)}^o = \underline{r}$, we can easily recover the actual model innovation ϵ_{t+1} from the biased model innovation $\hat{\epsilon}_{t+1}$; i.e., via the following decomposition.

$$\hat{\epsilon}_{t+1} = v_t + \epsilon_{t+1} \quad (14)$$

The corrected model is summarized in Table 4.

¹⁸Since the Fed gives out new forward guidance almost every month (Glick and Leduc (2013)), and such policy guidance is interpreted by the market participants as conveying important information about the Fed's policy reaction function (Femia, Friedman, and Sack (2013)). It is highly likely that the post-crisis FAVAR model has time-varying function forms.

This section defines v_t to be the bias generated by model mis-specification. Moreover, I show that a FAVAR model with time-varying parameters or time-varying function form is just a special case of the FGC-FAVAR model, and the FGC-FAVAR model proposed in this paper is more universal than it seems.

5.4 Estimation

For ease of exposition, we define constant column vector $e = [0, 0, 0, 1]'$, which is used to extract the policy rate s_t^o from z_t ; i.e., $s_t^o = e'z_t$.

We focus on the estimation of the autoregressive (5). Our estimation strategy is iterative:

1. For the pre-ZLB period (1960m1-2008m12), estimate (5) directly, setting $v_t = 0$ for all periods, get μ , ρ and Σ .
2. Set initial $t=2009m1$. Recall the market participants are able to infer/form an expectation of the ZLB duration $N(t)$ based on the Fed's period- t forward guidance, and we extract this $N(t)$ from survey data.

According to forward guidance, the expected policy rate at period $t + N(t)$ would be exactly \underline{r} ; i.e.,

$$E_t s_{t+N(t)}^o = \underline{r} \quad (15)$$

According to the traditional FAVAR model, the expected policy rate at period $t + N(t)$ would be

$$\hat{E}_t s_{t+N(t)}^o = e' \hat{E}_t z_{t+N(t)} = e' \left[\sum_{i=0}^{N(t)-1} \rho^i \mu + \rho^{N(t)} z_t \right] \quad (16)$$

Now we are able to calculate the bias d_t between the traditional FAVAR and the Fed's forward guidance.

$$d_t = E_t s_{t+N(t)}^o - \hat{E}_t s_{t+N(t)}^o = \underline{r} - e' \left[\sum_{i=0}^{N(t)-1} \rho^i \mu + \rho^{N(t)} z_t \right] \quad (17)$$

3. If $d_t = 0$, it means the period- t forward guidance information has already been properly captured, or that the period- t forward guidance information is negligible. Either way, we can set $v_t = 0$, and re-estimate Σ with ϵ_{t+1} ¹⁹, then go back to Step 2 and start at period $t+1$. If $d_t \neq 0$, we proceed to Step 4 to estimate the forward guidance term v_t .
4. Standing at period t , the market participants expect to receive forward guidance information $E_t v_{t+i}$ from the Federal Reserve for each period $t+i \in [t, t+N(t)-1]$, and the contribution of $E_t v_{t+i}$ at period $t+i$ on $E_t z_{t+N(t)}$ is $\rho^{N(t)-1-i} \Sigma \cdot E_t v_{t+i}$. Hence the overall effect of receiving forward guidance information $E_t v_{t+i}$ from the Federal Reserve for each period $t+i \in [t, t+N(t)-1]$ on $E_t z_{t+N(t)}$ would be

$$\left[\sum_{i=0}^{N(t)-1} \rho^{N(t)-1-i} \Sigma \cdot E_t v_{t+i} \right] \quad (18)$$

¹⁹To fully utilize the data, we estimate the Σ parameter recursively.

The corresponding effect on $E_t s_{t+N(t)}^o$ would be

$$e' \left[\sum_{i=0}^{N(t)-1} \rho^{N(t)-1-i} \Sigma \cdot E_t v_{t+i} \right] \quad (19)$$

v_t captures the effect of the past month's forward guidance on the level of the shadow rate. Since market participants do not expect forward guidance to change (the Fed is credible), I hold v_t constant from now to the expected lift-off date²⁰. That is, for $\forall t$, $E_t v_{t+1}^{MP} = v_t^{MP}$.

A “*guess and verify*” strategy would be helpful. That is, I am going to take the random walk assumption as given for now. And after I have estimated the FGC-FAVAR model, I will test the $\{v_t^{MP}\}_t$ series to see if it indeed follows a random walk process. As a sneak peek, the variance ratio test gives a p-value of 0.24, which indicates the random walk assumption is reasonable.

Following the random walk assumption, the overall forward guidance effect on $E_t s_{t+N(t)}^o$ (Equation (19)) can be simplified as $e' \left[\sum_{i=0}^{N(t)-1} \rho^{N(t)-1-i} \right] \Sigma v_t$.

5. Since v_t guarantees $E_t s_{t+N(t)}^o = \underline{r}$, we must have

$$e' \left[\sum_{i=0}^{N(t)-1} \rho^{N(t)-1-i} \right] \Sigma v_t = d_t \quad (20)$$

Note we have 1 unknown (v_t^{MP}), and (20) imposes 1 restriction. Thus we can estimate the $\{v_t\}_t$ ($\{v_t^{MP}\}_t$) series.

6. Re-estimate Σ with ϵ_{t+1} , and then go back to Step 2 and start at period $t+1$.

The estimation of (10) is trivial.

5.5 Unconventional Monetary Policy Innovations

Following the algorithm in Section 5.4, we are able to estimate the FGC-FAVAR model, and obtain the forward guidance term v_t which measures the information given out by period- t Federal Reserve's forward guidance, and the actual model innovations ϵ_{t+1} which describes unconventional monetary policy. I have argued previously that the traditional FAVAR model innovation $\hat{\epsilon}_{t+1}$ confounds the forward guidance term v_t with the model innovation ϵ_{t+1} , thus we would have the following decomposition:

$$\hat{\epsilon}_{t+1}^{MP} = v_t^{MP} + \epsilon_{t+1}^{MP} \quad (21)$$

Figure 7 compares (1) the biased money innovation $\{\hat{\epsilon}_t^{MP}\}_t$ estimated by the traditional FAVAR model (in blue), (2) the money innovation $\{\epsilon_t^{MP}\}_t$ estimated by the FGC-FAVAR model (in red), and

²⁰Similar pattern can be found from the Federal Reserve's pre-crisis conventional monetary policy. For example, if the Fed wanted to stimulate the economy, usually it would not simply lower the federal funds rate by 75 bps once and for all. Instead it is more likely that the Fed would lower the federal funds rate by 25 bps each month over the next quarter.

(3) the forward guidance term $\{v_{t-1}^{MP}\}_t$ estimated by the FGC-FAVAR model (in black). From the comparison, we can see that the forward guidance term $\{v_{t-1}^{MP}\}_t$ is generally non-zero, indicating that the forward guidance information is non-trivial, and that it indeed provides additional information to the market participants about future monetary policy. This finding is consistent with Femia, Friedman, and Sack (2013). Besides, we find that the actual monetary policy innovations $\{\epsilon_t^{MP}\}_t$ are significantly more negative and greater in absolute value than $\{\hat{\epsilon}_t^{MP}\}_t$, indicating that the traditional FAVAR model has severely underestimated the aggressiveness of the (unconventional) monetary policy in the post-crisis period. Put these observations together, and we can summarize the economic status quo during the ZLB period: On one hand, for a long time, the market always expects that lift-off is just around the corner (within a year or so). On the other hand, the Fed is trying to convince the market that the Fed has shifted to a more accommodative policy approach and the expected ZLB duration would be longer than the market expects. Since contemporaneous monetary policy and expected future monetary policy (expected lift-off date) are correlated in the real world, to convince the market, the Fed surprises the market month after month (contemporaneous money shocks). After the market finds that the contemporaneous money shock is more dovish than it expected, the market would then slowly defer its expected lift-off date.

This status quo can also be demonstrated from another perspective — the wordings of forward guidance announcements. During the ZLB period, the Federal Reserve keeps communicating with the public about the severity of the economic downturn and the resulting need for highly accommodative monetary policy for quite some time. The FOMC gradually made the forward guidance more explicit and forceful; for example, from its December 2008 statement’s “(the FOMC) . . . anticipates exceptionally low levels of the federal funds rate *for some time*” to its August 2011 statement’s “economic conditions . . . are likely to warrant exceptionally low levels for the federal funds rate *at least through mid-2013*” (see Table 2). Because of the Fed’s effort, the market participants’ expectation about the expected lift-off date was successfully guided from within a year or so to late 2013, and eventually guided to late 2015 (see Figure 5).

In sum, my exercise indicates that the market participants had always expected a tighter monetary policy during the ZLB period. The intuition is straightforward. When the economy enters a new world (ZLB), the Fed switches to a highly accommodative monetary policy, and the Fed tries hard to communicate its monetary stance with the market via forward guidance. As the time passes, the sluggish economy gradually calls for prolonged unconventional monetary policy. To convince the market of the continuous monetary easing (deferred lift-off date), the Fed has to surprise the market month after month. In face of the unprecedented economic environment (ZLB) and monetary policy (LSAP), the usual rational expectations rules probably do not apply.

5.6 Intervention Analysis

Having decomposed the biased traditional money innovation $\hat{\epsilon}_{t+1}$ into a forward guidance term v_t and the money innovation ϵ_{t+1} , I will conduct an intervention analysis in this section; i.e., run a counterfactual analysis that simulates the paths of major economic variables if we shut off unconventional monetary policy innovations $\{\epsilon_t^{MP}\}_t$ for the post-crisis period (July 2009 - December 2013). Then I compare my counterfactual paths with the constructed/observed economic variable paths, and the counterfactual paths where the money shocks are estimated in a traditional FAVAR framework.

In this benchmark simulation, as I shut down the monetary policy shocks $\{\epsilon_t^{MP}\}_t$, I am assuming

the Fed would still give out the same amount of information about future monetary policy; that is, I assume:

Assumption 1 *The forward guidance term v_t is independent of the state variable z_t .*

When the I shut down the monetary policy shock, the implied series of lift-off dates moves ahead. That is, the implied length of ZLB duration shortens. This is a natural assumption, since I am assuming a less accommodative monetary policy in the counterfactual world.

Before I compare the simulation paths with the constructed/realized paths, there are a few comments I would like to make about Assumption 1:

- First of all, Assumption 1 does not contradict the definition of v_t .

Recall that in Section 5.3, v_t is defined as the term that corrects the model mis-specification, and is known at period t :

$$v_t \equiv \Sigma^{-1}[f_t(z_t) - (\mu + \rho z_t)] \quad (22)$$

Now if the post-crisis evolution of $\{z_t\}_t$ follows a time-varying process:

$$f_t(z_t) = \mu_t + \rho z_t \quad (23)$$

Then by definition,

$$v_t \equiv \Sigma^{-1}[f_t(z_t) - (\mu + \rho z_t)] = \Sigma^{-1}(\mu_t - \mu) \quad (24)$$

which is indeed independent of the state variable z_t . And we use the forward guided market expected lift-off date to identify this time-varying term v_t .

- For a robustness check, I relax Assumption 1 and allow some correlation between the money innovations $\{\epsilon_t^{MP}\}_t$ and the forward guidance terms $\{v_t\}_t$ in Section 5.7. I find the results similar to the benchmark simulation in this section.

Figure 8 plots the counterfactual paths where the money innovations $\{\epsilon_t^{MP}\}_t$ are shut down. Comparing the counterfactual paths with the constructed/realized paths and a standard FAVAR model's simulation, we have the following observations:

Observation I: The constructed policy rate was lower than the counterfactual policy rate (black dashed line) at all times, indicating that the Fed adopted an expansionary policy during this period.

Observation II Without unconventional monetary policy, the market would have pushed the policy rate to 0.3% by December 2013 (as opposed to the actual -2.1%). Moreover, in the absence of unconventional monetary policy, in December 2013, the unemployment rate would have been 10.5%, significantly higher than the realized 6.7%, and consistent with Baumeister and Benati (2013). The industrial production index would have been 88.0, which would barely have recovered from the June 2009 level (83.8), and also much lower than the observed 101.8. Capital utilization would be 66.9%, 10.3% lower than what we observe. Housing starts would be 408,465 lower (590,535 vs. 999,000). This comparison shows that unconventional monetary policy is effective, consistent with the literature and our knowledge. Interestingly, the accommodative monetary policy during this period has not boosted real activity at the cost of high inflation. Instead, monetary policy shocks have contributed to decreasing the consumer price index by 5.6. Our result exhibits the same price

puzzle that has been discussed in earlier macro studies; e.g., Sims(1992), Eichenbaum(1992), and Wu and Xia (2016)²¹.

In sum, the two counterintuitive observations found in a traditional FAVAR model disappear under the FGC-FAVAR framework. The FGC-FAVAR model is able to estimate unconventional monetary policy in the reasonable direction, to show how devastating the crisis could have been, to demonstrate that unconventional monetary policy has been effective and powerful, and to confirm that previous traditional FAVAR methods have significantly underestimated the effect of unconventional monetary policy. The estimation of macroeconomic effect of unconventional monetary policy is also higher than the estimates reported in the literature, one explanation for the high estimation is the consistently expansionary monetary shocks (see Figure 7).

5.7 Robustness

This section discusses two robustness checks of the FGC-FAVAR model.

5.7.1 Correlated Money Innovation and Forward Guidance

In Section 5.6’s benchmark intervention analysis, I conducted a counterfactual analysis by shutting off the money innovations $\{\epsilon_t^{MP}\}_t$, while keeping the forward guidance terms $\{v_t^{MP}\}_t$ unchanged. This benchmark analysis assumes that the money innovations $\{\epsilon_t^{MP}\}_t$ and the forward guidance terms $\{v_t^{MP}\}_t$ are independent of each other (see Assumption 1). In this section, I will relax Assumption 1 and allow correlation between the money innovations $\{\epsilon_t^{MP}\}_t$ and the forward guidance terms $\{v_t^{MP}\}_t$. That is: in the benchmark analysis, the lift-off date implied by the FGC-FAVAR model was allowed to change (shorten) when the monetary shocks $\{\epsilon_{t+1}^{MP}\}_t$ were set to zero; in this section, however, I will see to it that the lift-off date implied by the FGC-FAVAR model stays unchanged.

In this alternative setup, the forward guidance term v_t becomes a state-dependent variable, contingent on the value of z_t . This alternative assumption is also intuitive to understand. The Federal Reserve gives out forward guidance information conditional on its understanding of the current (and expected future) economy. If we (or the Fed) stopped the monetary easing policy by shutting off ϵ_t^{MP} , the economic performance would not be as good, then the Fed would naturally give out less information about the incoming lift-off; i.e., the Fed would give out a small (less positive) forward guidance term v_t^{MP} . Since v_t^{MP} and ϵ_{t+1}^{MP} generally offset each other, the economic consequence of the counterfactual analysis in this setup is more complicated.

Recall that (the first 3 factors of) z_t summarizes the contemporary macroeconomic condition at period t , thus it is natural to allow for correlation between the state variable z_t and the forward guidance term v_t^{MP} . When we run a counterfactual analysis by shutting off the $\{\epsilon_t^{MP}\}_t$ series — which obviously changes the state variable vector z_t — we need to re-compute the new value of

²¹Bernanke, Boivin, and Eliasziw (2005) show that the FAVAR framework has already done a great job in reducing the price puzzle. To completely eliminate the price puzzle, ideally, one has to include “all” the macroeconomic variables that are likely to contain information relevant to the central bank’s decisions into the factor extraction step; i.e., include “all” the relevant variables into the dataset Y_t . This is an impossible mission, specially during the ZLB, when the Federal Reserve’s unconventional monetary policy is way more complicated to interpret and anticipate than the pre-crisis period. Besides, as a Ph.D. student, I really cannot afford to buy more data.

v_t along the simulation path by imposing the expected lift-off constraint²² $E_t s_{t+N(t)}^o = \underline{r}$. The computation algorithm still follows Section 5.4.

As a result, the counterfactual paths of the macroeconomic variables implied by the alternative intervention analysis setup resembles those implied by the benchmark setup, see Figure 9. There is some difference between the black line (my benchmark counterfactual paths with independent exogenous $\{v_t\}_t$ series) and the green line (the new counterfactual paths with endogenous $\{v_t\}_t$ series). However, the dynamics of the two sets of series exhibit a strong co-movement, with a correlation of 0.89 for the industrial production, 1.00 for the CPI, 0.91 for the capital utilization, 0.95 for the unemployment rate, and 0.92 for the housing starts.

More importantly, they produce similar qualitative economic implications. Our key result in Section 5.6 holds. Similar to the benchmark case, we find that:

- The constructed policy rate had been lower than the counterfactual policy rate at all times, indicating that the Fed had been adopted an expansionary policy during this period. This is consistent with our knowledge.
- Without the money shocks, the market would push the policy rate to 2.3% by December 2013. Moreover, in the absence of unconventional monetary policy, in December 2013, the unemployment rate would be 11.0%, significantly higher than the realized 6.7%. The industrial production index would have been 84.2, lower than the observed 101.8, and has barely recovered from the June 2009 level (83.8). Capital utilization would be 64.8%, 12.3 percentage points lower than what we observe. Housing starts would be 471,561 lower (527,439 vs. 999,000). This comparison shows that unconventional monetary policy is effective, consistent with the literature and our knowledge.

In sum, whether we keep the forward guidance term $\{v_t\}_t$ exogenous, as in our benchmark setup, or estimate it as an endogenous term does not alter any conclusion. The conclusion in this paper is robust with respect to Assumption 1.

5.7.2 Different Ways of Using Survey Information

In my benchmark simulation, I extracted the expected lift-off date series $\{t + N(t)\}_t$ from the survey data, imposed the restriction $E_t s_{t+N(t)}^o = \underline{r}$, and estimated a FGC-FAVAR model. In this section, I try different ways of using the information in the survey. I first calculate an average expected ZLB duration $\bar{N} = \frac{\sum_t N(t)}{T} = 16$ months, and then impose an alternative restriction that for $\forall t$, $E_t s_{t+\bar{N}} = \underline{r}$.

Using this alternative restriction, I am able to re-estimate the whole FGC-FAVAR model, including the forward guidance term $\{v_t\}_t$ series and the model innovation $\{\epsilon_{t+1}\}_t$ series. Also we are able to conduct the intervention analysis, and compare it with the benchmark result in Section 5.6.

²²The reader might argue that a more reasonable assumption to introduce correlation between $\{v_t^{MP}\}_t$ and $\{z_t\}_t$ would be to assume a direct relationship between the economic condition $\{z_t\}_t$ and the expected lift-off date $\{t + N(t)\}_t$, rather than imposing a purely exogenous series $\{t + N(t)\}_t$. The difficulty is, so far, no qualitative relationship between $\{z_t\}_t$ and $\{t + N(t)\}_t$ has ever been found. Like I said in Section 2, unconventional monetary policy is unprecedented, which brings a lot of uncertainty, and makes the Fed extremely cautious in this process. We have reasons to believe that the Fed is improvising, rather than operating with an instruction manual.

Figure 10 shows the counterfactual paths (in green) of the macroeconomic variables implied by the FGC-FAVAR model with average ZLB duration \bar{N} , and compares them with the benchmark counterfactual paths (in black). We find that all the counterfactual paths resemble those implied by the benchmark setup, indicating the robustness of the FGC-FAVAR model.

6 Conclusion

The literature has estimated a shadow rate to summarize the overall stance of unconventional monetary policy. However, due to omitting the rest of the information contained in the term structure, the traditional FAVAR model that uses the shadow rate as the only policy measure delivers biased unconventional monetary policy innovation and underestimated macroeconomic effect.

To address this problem, I develop a FGC-FAVAR model which extracts the most representative information — expected lift-off date — from the market survey, and incorporates this forward guided expectation into the FAVAR model. Imposing this information as a restriction on the dynamic system, I successfully decomposed the (biased) FAVAR model innovation $\hat{\epsilon}_{t+1}$ into two components: (1) a forward guidance term v_t , which is known at period t , but does not “hit” until period $t + 1$. This v_t term represents the information given out by the Fed through forward guidance, and gives the market participants a better expectation about future monetary policy. (2) The remaining unknown component is the actual model innovation ϵ_{t+1} .

Using the newly estimated model innovation ϵ_{t+1} , I conduct intervention analysis by shutting off the money innovation during the post-crisis period (July 2009 - December 2013), and find two observations: (1) The Fed had adopted an expansionary policy during this period. (2) Unconventional monetary policy is more aggressive and effective than the traditional FAVAR model has previously estimated, and the policy effect I estimated is generally larger than the literature would suggest. This is probably due to the fact that I properly controlled the market participants’ anticipation about future monetary policy.

The FGC-FAVAR model setup is generic and inclusive. The additive forward guidance term is consistent with various forms of forward guidance information. For example, a FAVAR model with time-varying parameters or time-varying function forms can be directly mapped into the FGC-FAVAR framework.

I also test the robustness of the FGC-FAVAR model by (1) introducing correlation between the forward guidance term v_t and the model innovation ϵ_{t+1} , and (2) estimating the FGC-FAVAR model by using the survey information in a different way. I demonstrate the robustness of the FGC-FAVAR model against these alternative setups, and find the main conclusions of this paper are not altered.

A Appendix

A.1 Extract Lift-off Date $\{t + N(t)\}_t$ from Survey Data

This paper uses two survey datasets, both have pros and cons:

- Bluechip Financial Forecast (Bluechip):

- pros — monthly survey data
- cons — only contains expectation up to 6 quarters ahead
- New York Fed Primary Dealer survey (NYPDS):
 - pros — can contain expectation more than 6 quarters
 - cons — the dataset does not start until January 2011, and the survey is conducted only 8 times per year (on the month of FOMC meetings), so it is less frequent than monthly, but more frequent than quarterly.

I mainly use Bluechip, because (i) Bluechip survey is conducted monthly, and (ii) NYPDS does not start until January 2011.

Only since late 2011 when the expected lift-off date is more than 6 quarters ahead, which exceeds Bluechip’s survey range, then I switch to NYPDS. This methodology to extract and splice the expected lift-off date is exactly the same as Williams (2014).

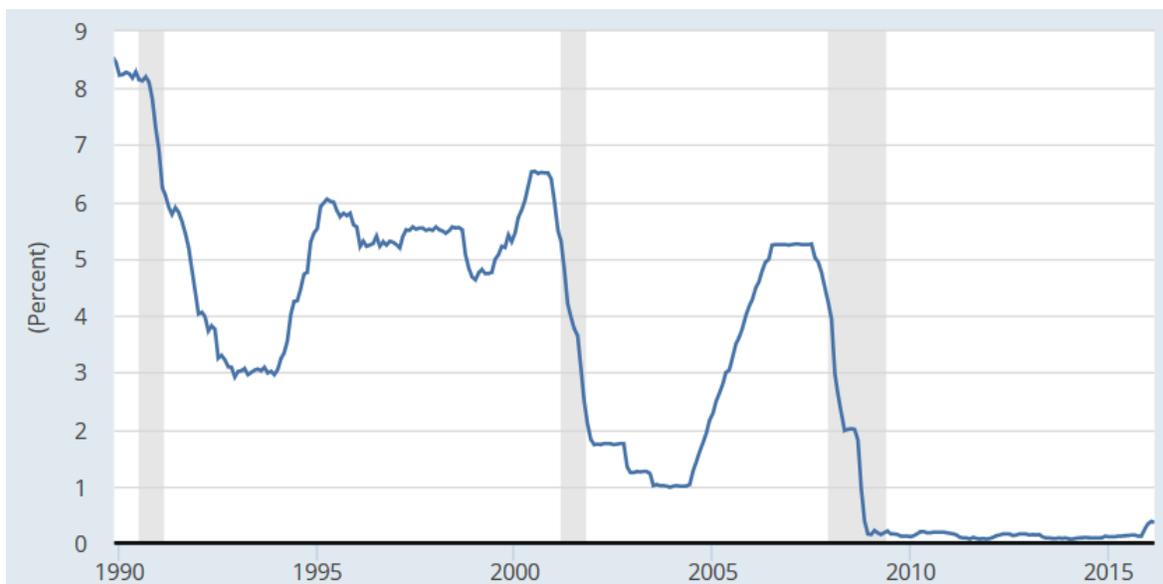
In all, there are only a few discrete occasions/months that do not have a survey expected lift-off date. Given that no FOMC meetings were held during these months (which indicates that comparatively less forward guidance information was released), I assume these months have exactly the same lift-off dates as that of the latest previous month.

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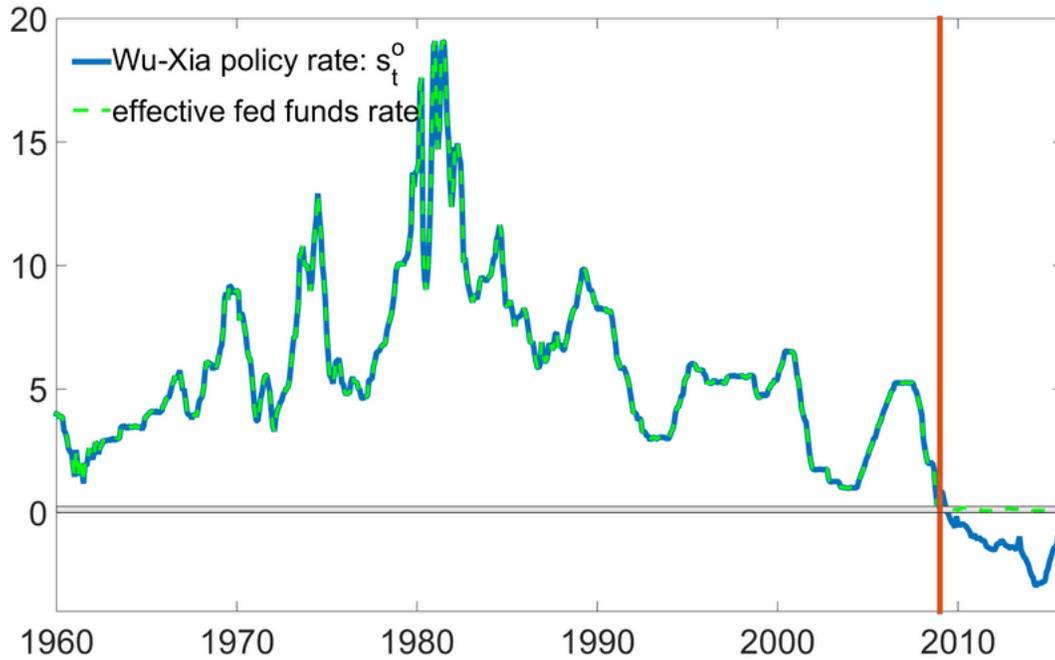
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Figure 1: Effective Federal Funds Rate



Source: Board of Governors of the Federal Reserve System, retrieved from FRED, Federal Reserve Bank of St. Louis, <https://research.stlouisfed.org/fred2/series/FEDFUNDS>.

Figure 2: Shadow Rate by Wu and Xia (2016)



Green line: effective federal funds rate. Blue line: policy rate constructed by splicing the effective federal funds rate before January 2009 with Wu and Xia (2016)'s shadow rate since January 2009.

Figure 3: Sample Survey Question for Expected Lift-off Date

Of the possible outcomes below, please indicate the percent chance you attach to the timing of the first federal funds target rate increase.

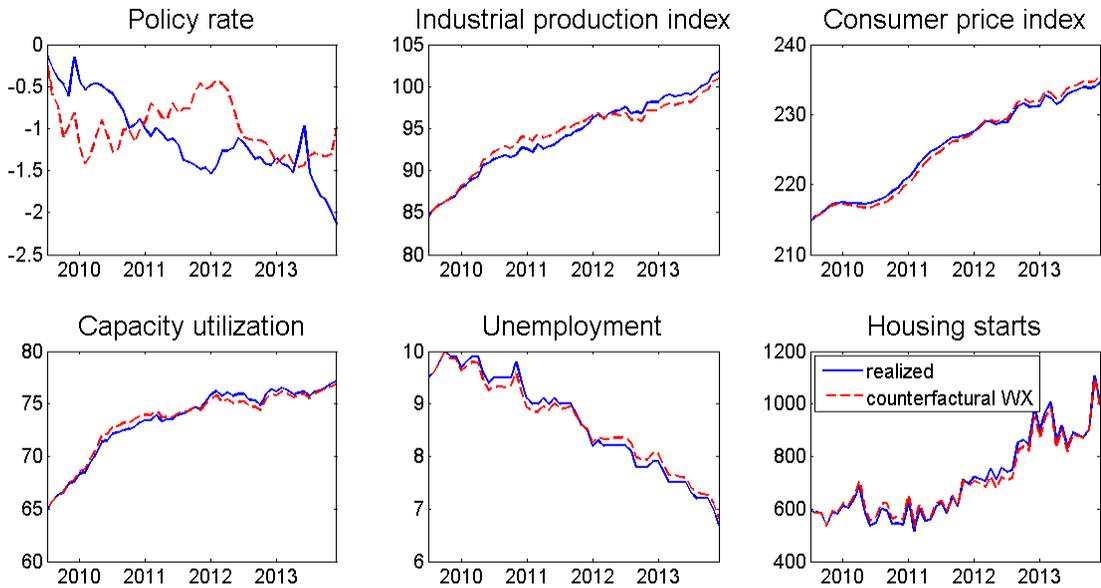
	H1 2012	H2 2012	H1 2013	H2 2013	H1 2014	H2 2014	H1 2015	H2 2015	H1 2016	≥ H2 2016
Average	0%	1%	4%	12%	19%	26%	15%	10%	8%	6%

Most likely quarter and year of first target rate increase:	
25th Pctl	Q2 2014
Median	Q3 2014
75th Pctl	Q4 2014

This is a sample survey question for the expected lift-off date from the New York Fed's Primary Dealer Survey for March, 2012.

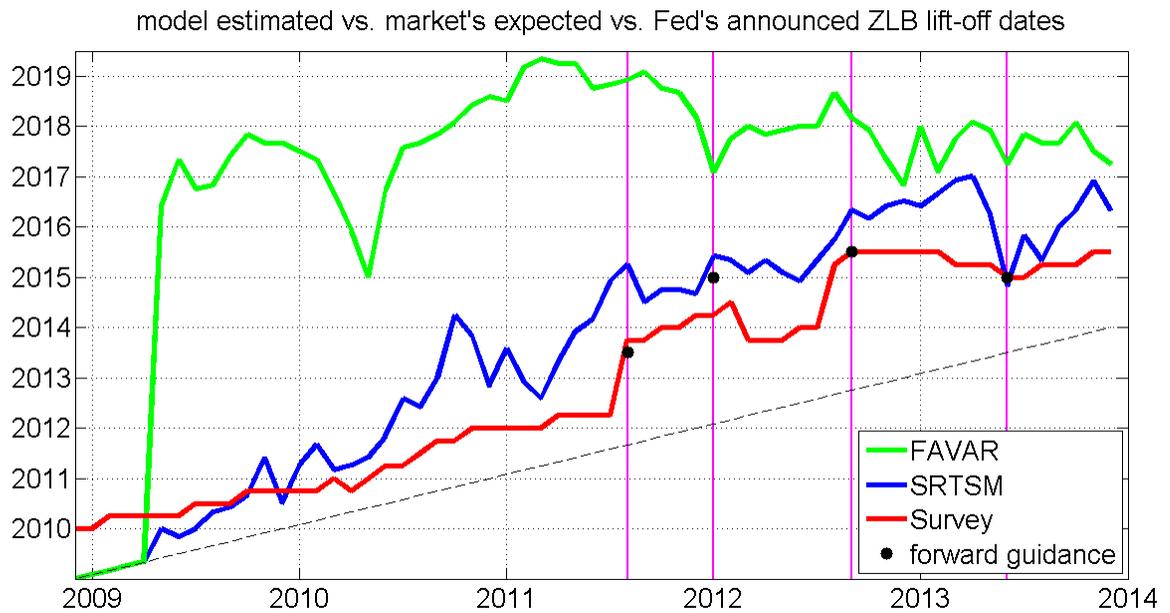
Source: https://www.newyorkfed.org/medialibrary/media/markets/survey/March_2012_result.pdf

Figure 4: Wu and Xia (2016): Constructed/Observed vs. Counterfactual Macroeconomic Variables



Blue lines: observed economic variables between July 2009 and December 2013, the policy rate is constructed from Wu and Xia (2016)’s term structure model. Red dashed lines: what would have happened to these macroeconomic variables, if all the monetary policy shocks were shut down.

Figure 5: Expected Lift-off Date: FAVAR vs. SRTSM vs. Survey



Green line: the expected lift-off date $\{t + \hat{N}(t)\}_t$ series predicted by the traditional FAVAR model from January 2009 to December 2013. Blue line: the expected lift-off date $\{t + \tilde{N}(t)\}_t$ series predicted by the shadow rate term structure model. Red line: the survey data series on expected lift-off date $\{t + N(t)\}_t$ by Blue Chip Financial Forecast and the New York Fed's Primary Dealer Survey. Four purple vertical lines mark the following months when forward guidance specified explicit lift-off dates for the ZLB: August 2011, January 2012, September 2012 and June 2013. The corresponding lift-off dates are in black dots. Black dashed line: the 45 degree line.

Figure 6: Economic Logic for Incorporating Expected Lift-off Date

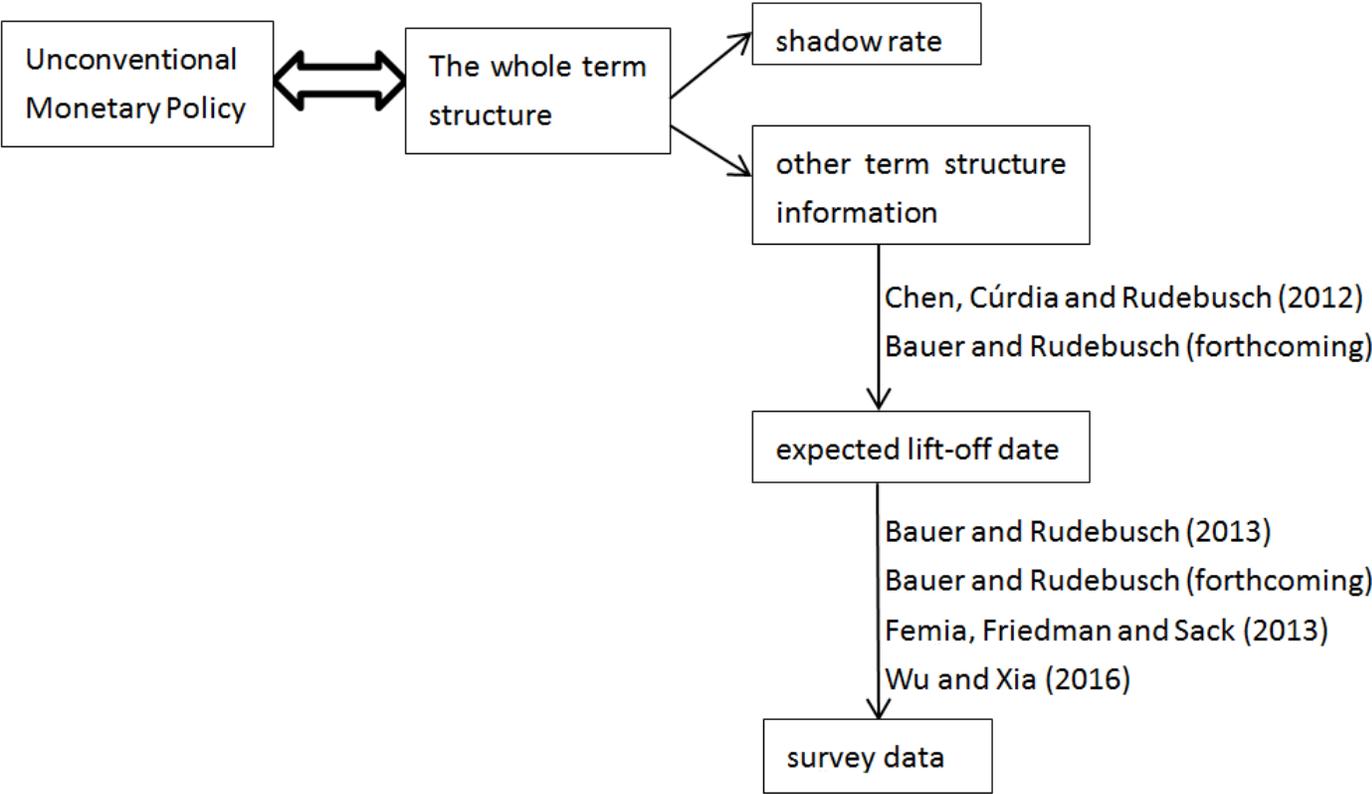
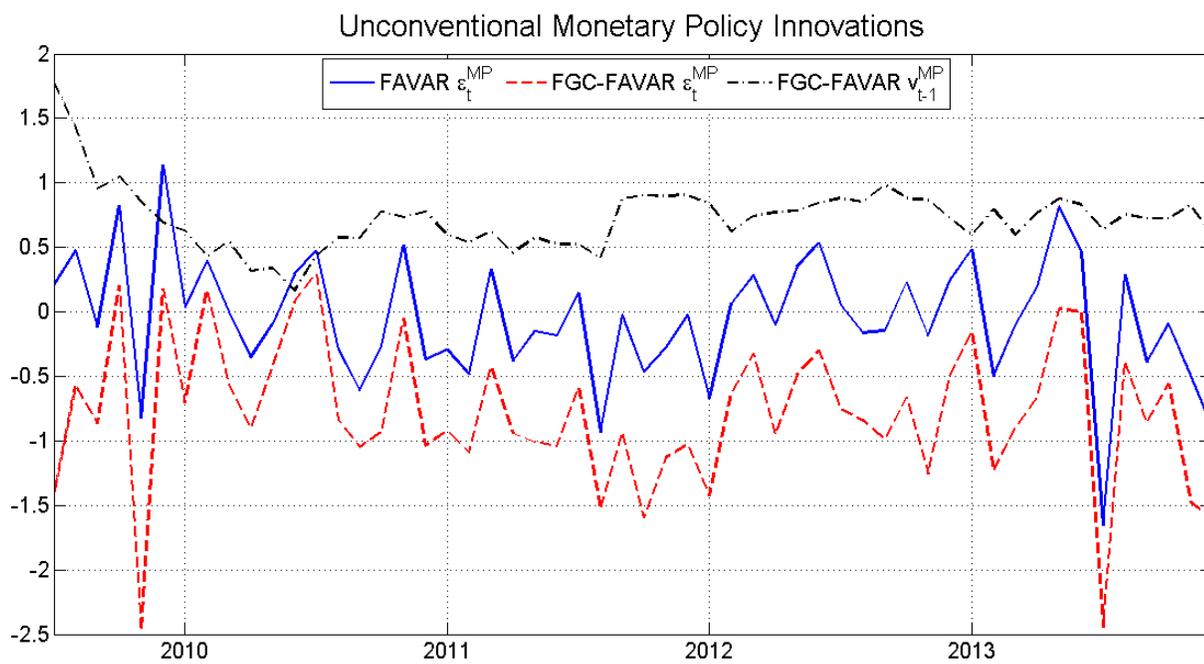
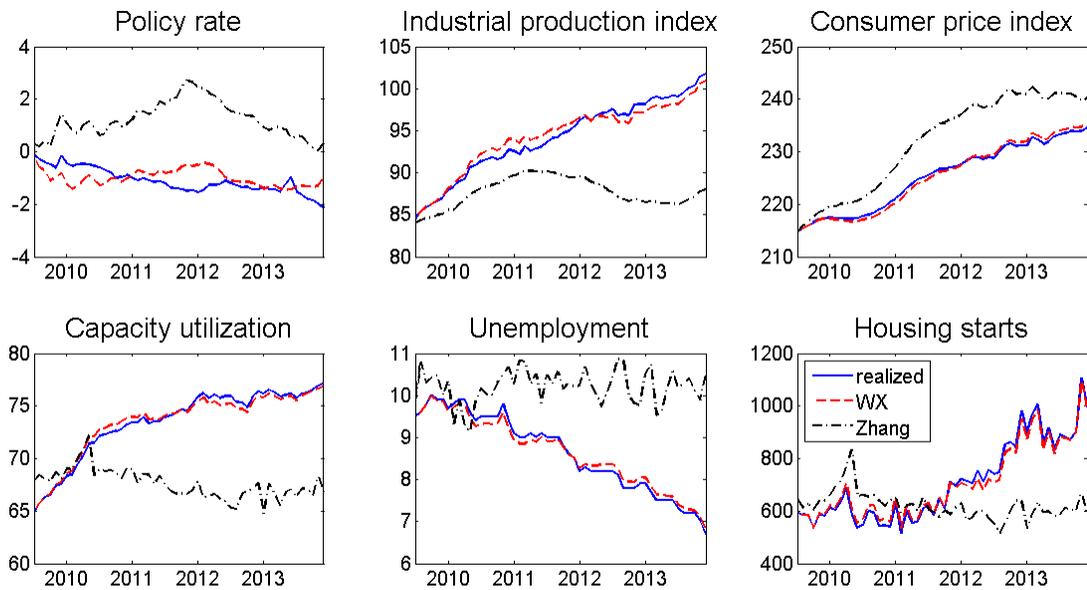


Figure 7: Unconventional Monetary Policy Innovations



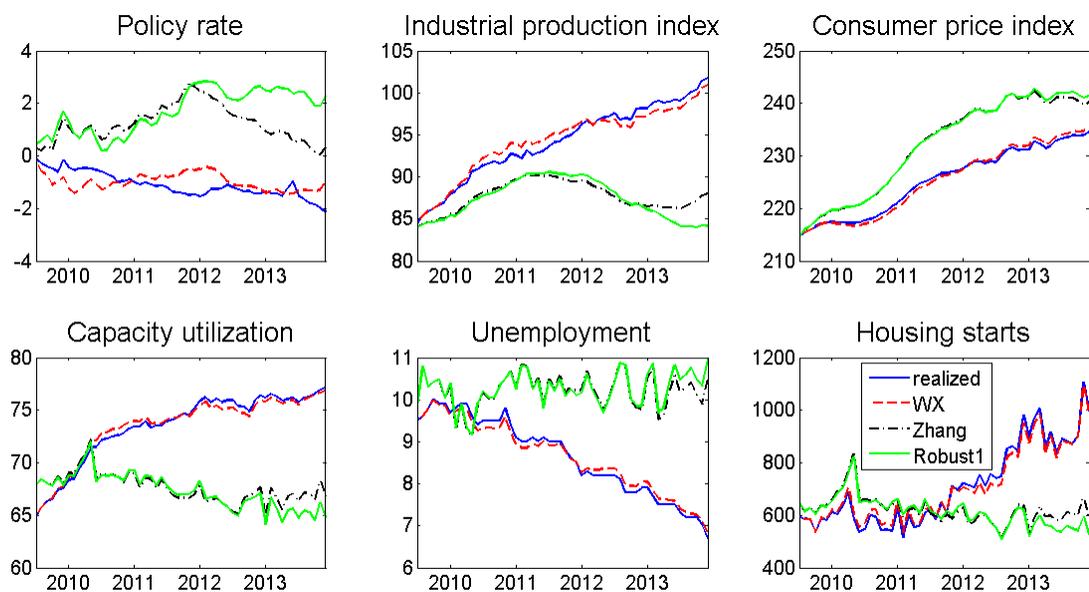
Blue line: the money innovation $\hat{\epsilon}_t^{MP}$ estimated by the traditional FAVAR model from July 2009 to December 2013. Red line: the money innovation ϵ_t^{MP} estimated by the FGC-FAVAR model from July 2009 to December 2013. Black line: the forward guidance term v_{t-1}^{MP} estimated by the FGC-FAVAR model from July 2009 to December 2013.

Figure 8: FGC-FAVAR: Intervention Analysis



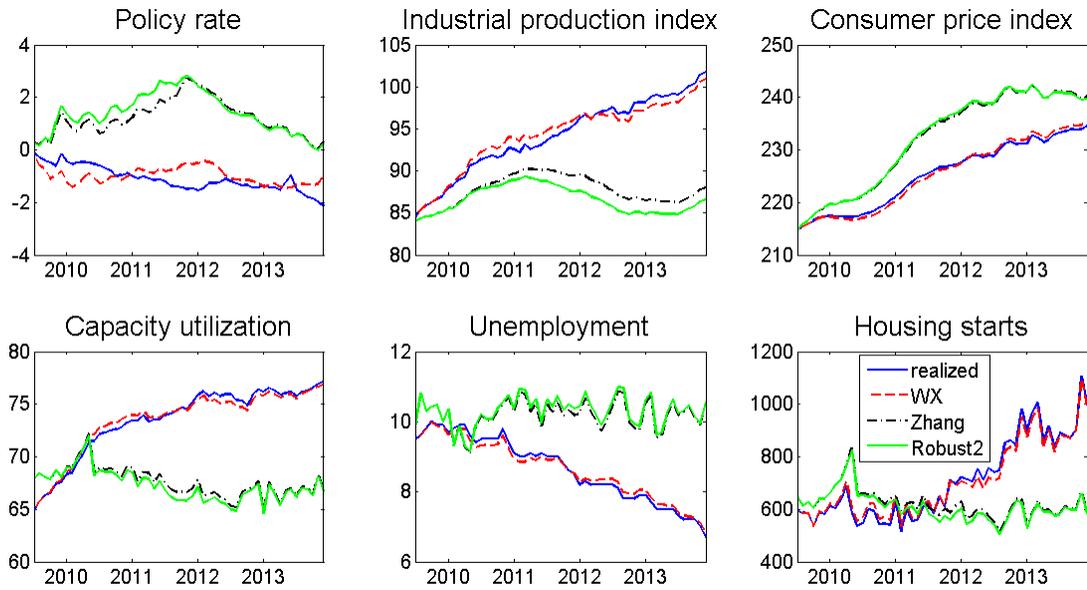
Blue lines: observed economic variables between July 2009 and December 2013, the policy rate is constructed from Wu and Xia (2016)'s term structure model. Red lines: FAVAR model's intervention analysis if all the monetary policy shocks were shut down. Black lines: FGC-FAVAR model's intervention analysis if all the monetary policy shocks were shut down.

Figure 9: FGC-FAVAR: Intervention Analysis (endogenous v_t)



Blue lines: observed economic variables between July 2009 and December 2013, the policy rate is constructed from Wu and Xia (2016)'s term structure model. Red lines: Wu and Xia (2016) intervention analysis if all the monetary policy shocks were shut down. Black lines: FGC-FAVAR model's benchmark intervention analysis if all the monetary policy shocks were shut down. Green lines: FGC-FAVAR model's intervention analysis (endogenous v_t) if all the monetary policy shocks were shut down.

Figure 10: FGC-FAVAR: Intervention Analysis (Average ZLB Duration)



Blue lines: observed economic variables between July 2009 and December 2013, the policy rate is constructed from Wu and Xia (2016)'s term structure model. Red lines: Wu and Xia (2016) intervention analysis if all the monetary policy shocks were shut down. Black lines: FGC-FAVAR model's benchmark intervention analysis if all the monetary policy shocks were shut down. Green lines: FGC-FAVAR model's intervention analysis (with average ZLB duration) if all the monetary policy shocks were shut down.

Table 1: Large Scale Asset Purchase Programs

	Date	Content
QE1	11/2008-03/2010	buy \$600 billion in mortgage-backed securities, later added an additional \$750 billion purchases of agency MBS and agency debt, and \$300 billion purchases of Treasury securities. By March 2009, the Fed held \$1.75 trillion of bank debt, mortgage-backed securities, and Treasury notes; this amount reached a peak of \$2.1 trillion in June 2010
QE2	11/2010-06/2011	buy \$600 billion of Treasury securities by the end of the second quarter of 2011
Operation Twist	09/2011-12/2012	purchase \$400 billion (later added an additional \$267 billion) of bonds with maturities of 6 to 30 years and to sell bonds with maturities less than 3 years, thereby extending the average maturity of the Fed's own portfolio.
QE3	09/2012-10/2014	purchase \$40 billion agency mortgage-backed securities per month. Since 12/12/2012, authorize up to \$40 billion worth of agency mortgage-backed securities per month and added \$45 billion worth of longer-term Treasury securities. Since 12/18/2013, start to taper back at a rate of \$10 billion at each meeting.

Source: Board of Governors of the Federal Reserve System

Table 2: Forward Guidance Quotes

Date	Quotes
12/16/2008	“... anticipates that weak economic conditions are likely to warrant exceptionally low levels of the federal funds rate for <i>some time</i> .”
03/18/2009	“... anticipates that economic conditions are likely to warrant exceptionally low levels of the federal funds rate for <i>an extended period</i> .”
08/09/2011*	“... anticipates that economic conditions — including low rates of resource utilization and a subdued outlook for inflation over the medium run — are likely to warrant exceptionally low levels for the federal funds rate <i>at least through mid-2013</i> .”
01/25/2012*	“... decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that economic conditions — including low rates of resource utilization and a subdued outlook for inflation over the medium run — are likely to warrant exceptionally low levels for the federal funds rate <i>at least through late 2014</i> .”
09/13/2012*	“... decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that exceptionally low levels for the federal funds rate are likely to be warranted <i>at least through mid-2015</i> .”
12/12/2012	“... decided to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that this exceptionally low range for the federal funds rate will be appropriate at least as long as the unemployment rate remains above 6-1/2 percent, inflation between one and two years ahead is projected to be no more than a half percentage point above the Committee’s 2 percent longer-run goal, and longer-term inflation expectations continue to be well anchored.”
06/19/2013*	“... 14 of 19 FOMC participants indicated that they expect the first increase in the target for the federal funds rate to occur in <i>2015</i> , and one expected the first increase to incur in 2016.”
12/18/2013	“... anticipates, based on its assessment of these factors, that it likely will be appropriate to maintain the current target range for the federal funds rate well past the time that the unemployment rate declines below 6-1/2 percent, especially if projected inflation continues to run below the Committee’s 2 percent longer-run goal.”

This table summarizes a list of forward guidance quotes, when the Fed talks about expected lift-off date or condition for the ZLB. All quotes except the one on 06/19/2013 are from FOMC statements. The quote on 06/19/2013 is from Chairman Bernanke’s press conference. Asterisks mark the statements with explicit lift-off dates, with the corresponding lift-off dates in red.

Source: <http://www.federalreserve.gov/monetarypolicy/fomccalendars.htm>

Table 3: mistakenly use the pre-crisis FAVAR model to fit the post-crisis data

	mis-specified model (FAVAR)	actual model
Model	$z_{t+1} = \mu + \rho z_t + \Sigma \hat{\epsilon}_{t+1}$	$z_{t+1} = f_t(z_t) + \Sigma \epsilon_{t+1}$
Expectation	$\hat{E}_t z_{t+1} = \mu + \rho z_t$	$E_t z_{t+1} = f_t(z_t)$
Residual	$\Sigma \hat{\epsilon}_{t+1} = [f_t(z_t) - (\mu + \rho z_t)] + \Sigma \epsilon_{t+1}$	$\Sigma \epsilon_{t+1}$
Innovation	$\hat{\epsilon}_{t+1} = \Sigma^{-1} [f_t(z_t) - (\mu + \rho z_t)] + \epsilon_{t+1}$	ϵ_{t+1}

Table 4: use the FGC-FAVAR model to recover the model innovation

	mis-specified model	actual/corrected model
Model	$z_{t+1} = \mu + \rho z_t + \Sigma \hat{\epsilon}_{t+1}$	$z_{t+1} = f_t(z_t) + \Sigma \epsilon_{t+1}$ $= (\mu + \rho z_t) + \Sigma v_t + \Sigma \epsilon_{t+1}$
Expectation	$\hat{E}_t z_{t+1} = \mu + \rho z_t$	$E_t z_{t+1} = f_t(z_t)$ $= \mu + \rho z_t + \Sigma v_t$
Residual	$\Sigma \hat{\epsilon}_{t+1} = \Sigma v_t + \Sigma \epsilon_{t+1}$	$\Sigma \epsilon_{t+1}$
Innovation	$\hat{\epsilon}_{t+1} = v_t + \epsilon_{t+1}$	ϵ_{t+1}